

# A mixed damage model for unsaturated porous media

## Un modèle d'endommagement mixte pour les milieux poreux non saturés

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### Abstract

The aim of this study is to present a framework for the modeling of damage in continuous unsaturated porous geomaterials. The damage variable is a second-order tensor. The model is formulated in net stress and suction independent state variables. Correspondingly, the strain tensor is split into two independent thermodynamic strain components. The proposed frame mixes micro-mechanical and phenomenological approaches. On the one hand, the effective stress concept of Continuum Damage Mechanics is used in order to compute the damaged rigidities. On the other hand, the concept of equivalent mechanical state is introduced in order to get a simple phenomenological formulation of the behavior laws. Cracking effects are also taken into account in the fluid transfer laws.

Cette étude a pour objectif de présenter un cadre théorique pour la modélisation de l'endommagement dans les géomatériaux non-saturés, considérés comme des milieux continus. Le modèle est formulé en variables d'état indépendantes : contrainte nette et succion. Conjointement, le tenseur des déformations est écrit comme la somme de deux composantes thermodynamiques indépendantes. Le cadre théorique proposé combine les approches micro-mécanique et phénoménologique. D'une part, le concept de contrainte effective de la Mécanique de l'Endommagement en Milieu Continu est utilisé, de manière à calculer les rigidités endommagées. D'autre part, le concept d'état mécanique équivalent est introduit pour obtenir une formulation phénoménologique simple des lois de comportement. Les effets de la fissuration sont également pris en compte dans les lois de transfert des fluides.

**Key words:** damage, poro-elasticity, geomaterial, net stress, suction, micro-mechanics, thermodynamics, permeability  
endommagement, poro-élasticité, géomatériau, contrainte nette, succion, micro-mécanique, thermodynamique, perméabilité

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## 1. Introduction

This study is motivated by the necessity to predict the behavior of the Excavation Damaged Zone surrounding nuclear waste disposals. The geological barriers, often made of quasi-brittle material like granite or clay-rock, undergo damage during the excavation phase. Hydro-mechanical interactions may occur in the neighborhood of the engineered barrier, which is generally made of unsaturated compacted clay. Fluids flow inside the gallery. The span of the transient regime depends on the relative values of the involved conductivities. The complex couplings involved in the geological massif before the waste disposal make it necessary to study rock-like quasi-brittle materials as multi-phase media. Up to now, almost all the damage models dedicated to non-dry materials have been formulated by means of a concept of effective stress. This choice is questionable for unsaturated soils [1]. That is why the present paper aims at developing a fully coupled damage model formulated in independent stress state variables, in order to study the behavior of cracked unsaturated porous geomaterials. Section 2 presents the physical and mathematical representations of damage. Section 3 is devoted to the phenomenological aspects of the model. The thermodynamic split of the strain tensor is explained, and the stress/strain relations are derived from a postulated expression of the free energy. This latter is chosen in accordance with the representation of damage exposed in section 2. The micro-mechanical developments of the proposed damage model are presented in section 4. The damaged rigidities are computed, and damaged transfer rules are established.

## 2. Damage representation

### 2.1. Micro-mechanical meaning of the damage tensor

It is assumed that the studied Representative Elementary Volume (REV) contains a network of non-interacting micro-cracks. If the REV is damaged by  $N$  micro-cracks, variation of the elastic deformation energy is:

$$\Delta W^e = \frac{1}{2V_{REV}} \sum_{k=1}^N \mathbf{n}^k \cdot \boldsymbol{\sigma} \cdot \langle \mathbf{b}^k \rangle S^k \quad (1)$$

in which  $S^k$  and  $\mathbf{n}^k$  are the surface of the  $k^{th}$  crack and the normal vector of the  $k^{th}$  crack plane, respectively.  $V_{REV}$  is the volume of the Representative Element,  $\sigma_{ij}$  is Cauchy stress tensor, and  $\langle \mathbf{b}^k \rangle$  is the  $k^{th}$  crack opening displacement. In three dimensions, crack openings have two shearing components. The corresponding micro-crack compliance tensors thus encompass fourth-order terms. However, the shearing contribution to the crack compliance tensor may be neglected [2]. As advocated by Kachanov, the second-order crack density tensor will be used in order to represent damage in the proposed model. This tensor may be expressed in its principal base as:

$$\Omega_{ij} = \sum_{k=1}^3 d^k n_i^k n_j^k \quad (2)$$

Stress and damage are assumed to have the same principal directions. Physically, the damaged behavior of the REV is modeled by three meso-cracks representing three main families of fissures. Each meso-crack is characterized by a direction  $\mathbf{n}^k$  (normal to the crack plane) and a

volumetric fraction  $d^k$ . Assuming that the cracks are penny-shaped, with a radius  $l^k$  and a thickness  $e^k$ :

$$d^k = \frac{1}{V_{REV}} e^k \pi (l^k)^2 \quad (3)$$

The radius and thickness of cracks may be related by a linear dilatancy rule [3] of the type:  $de^k = \chi dl^k$ , with  $\chi = 0.005$  for brittle rocks.

## 2.2. Equivalent mechanical state of the damaged material

The proposed model is dedicated to unsaturated quasi-brittle geomaterials, such as granite or clay rock. In such materials, frictional sliding induces crack opening. Gouge may be produced. the resulting asperities generate residual strains, remaining after unloading [4]. Only mode I failures will be investigated in the following frame. It is thus assumed that damage grows with tensile strains. Such an assumption on the physical origin of damage enables the description of the *splitting effects* due to traction, and of the *crossing effects* due to compression [5]. As recalled by Swoboda and Yang [4], the concentrated forces  $\tau_i^k$  opening the  $k^{th}$  tensile micro-crack may be represented microscopically by a stress tensor  $\tilde{\tau}_{ij}^k$ . This latter representation is named *equivalent mechanical state* by the authors. In the equivalent mechanical state, the micro-cracks are open, and the REV is subjected to an equivalent stress, defined as:

$$\sigma^{eq} = \sigma + \sum_{k=1}^N \tilde{\tau}^k = \sigma + \tilde{\tau} \quad (4)$$

in which  $\sigma$  is the far-field stress.  $\tilde{\tau}$  is the “homogenized crack-related stress”. Since the cracks are assumed to grow with tensile strains, the homogenized crack-related stress tensor  $\tilde{\tau}$  and the homogenized damage tensor  $\Omega$  are supposed to have the same principal directions, so that  $\tilde{\tau} = g \Omega$ . With the general mechanics convention,  $g$  is positive and has the dimension of a rigidity. In absence of external load,  $\sigma$  is null, and the damaged material is thus subjected to a residual equivalent traction, generating residual tensile strains. The concept of equivalent mechanical state thus enables the introduction of residual phenomena which are only due to damage, without using the concept of plasticity. This provides a relative simple theoretical frame, encompassing only one dissipative potential.

## 3. Phenomenological aspects of the damage model

### 3.1. Independent state variables

The pores of the solid matrix are assumed to be filled by two continuous fluid phases: liquid water and gaseous air. The behavior law is formulated in independent stress state variables [1]. Net stress  $\sigma''_{ij}$  is defined as the difference between the total Cauchy stress tensor  $\sigma_{ij}$  and the isotropic air pore pressure  $p_a \delta_{ij}$ :  $\sigma''_{ij} = \sigma_{ij} - p_a \delta_{ij}$ .  $\delta_{ij}$  denotes the second-order identity tensor. Suction is the difference between air and water pore pressures:  $s = p_a - p_w$ . According to the notations adopted in the models of Gatmiri [6, 7], the incremental behavior law is expressed in the following form:

$$d\sigma''_{ij} = D_{eijkl} (\Omega_{pq}) d\epsilon_{Mik}^e = D_{eijkl} (\Omega_{pq}) (d\epsilon_{Mik} - d\epsilon_{Mik}^d) \quad (5)$$

$\epsilon_{Mij}$  is the mechanical strain tensor, which is thermodynamically conjugated to net stress. It encompasses an elastic part  $\epsilon_{Mij}^e$  and an inelastic part  $\epsilon_{Mij}^d$ . The same approach is used to define a

capillary strain  $\epsilon_{S_{ij}}$ , thermodynamically conjugated to suction. Pore pressure effects are assumed to be isotropic, so that only the knowledge of the volumetric capillary strains is necessary to characterize the geomaterial behavior [6, 7]:  $\epsilon_{S_{ij}} = \frac{1}{3}\epsilon_{S_v}\delta_{ij}$ . Volumetric capillary strains have also an elastic ( $\epsilon_{S_v}^e$ ) and an inelastic ( $\epsilon_{S_v}^d$ ) component. The total incremental strain tensor is finally split as follows:

$$d\epsilon_{ij} = d\epsilon_{M_{ij}}^e + d\epsilon_{M_{ij}}^d + \frac{1}{3}\epsilon_{S_v}^e\delta_{ij} + \frac{1}{3}\epsilon_{S_v}^d\delta_{ij} \quad (6)$$

Assuming that both liquid and gaseous phases saturate the pores of the matrix, and adopting the standard notations, the Inequality of Clausius-Duhem writes:

$$\sigma_{ij}d\epsilon_{ji} + p_w d(nS_w) + p_a d(n(1 - S_w)) - d\Psi_s(\epsilon_{pq}, nS_w, n(1 - S_w), \Omega_{pq}) \geq 0 \quad (7)$$

in which  $n$  is the porosity,  $S_w$  is the saturation degree of the liquid phase and  $\Psi_s$  is Helmholtz free energy. With the notations adopted in this paper, equation 7 writes:

$$\sigma_{ij}d\epsilon_{M_{ji}} - p_w d\epsilon_{S_v} + p_a d(n + \epsilon_{S_v}) - d\Psi_s(\epsilon_{M_{pq}}, \epsilon_{S_v}, n + \epsilon_{S_v}, \Omega_{pq}) \geq 0 \quad (8)$$

The condition of solid grain incompressibility writes  $d\epsilon_{M_v} = -dn$ , so that the expression of the ICD used in the following is:

$$\sigma''_{ij}d\epsilon_{M_{ji}} + s d\epsilon_{S_v} - d\Psi_s(\epsilon_{M_{pq}}, \epsilon_{S_v}, \Omega_{pq}) \geq 0 \quad (9)$$

In elasticity, the combination of equations 5 and 6 leads to:

$$d\sigma''_{ij} = D_{e_{ijkl}}(\Omega_{pq}) \left( d\epsilon_{lk}^e - \frac{1}{3}\delta_{lk}d\epsilon_{S_v}^e \right) \quad (10)$$

in which capillary strains and suction are related by a damage-dependent capillary rigidity:

$$d\epsilon_{S_v}^e = \frac{ds}{\beta_s(\Omega_{pq})} \quad (11)$$

Therefore, the elastic incremental behavior law 10 may be expressed as:

$$d\sigma''_{ij} = D_{e_{ijkl}}(\Omega_{pq}) \left( d\epsilon_{lk}^e - \frac{1}{3\beta_s(\Omega_{pq})}\delta_{lk}ds \right) \quad (12)$$

which induces a coupling in the state equations.

### 3.2. Stress/strain relations

In the equivalent mechanical state described in section 2, the equivalent stress variables are conjugated to the strain state variables by means of a degraded elastic potential  $\Psi_e$  [4]:

$$\begin{cases} \sigma''_{ij}{}^{eq} = \frac{\partial \Psi_e(\epsilon_{M_{pq}}, \epsilon_{S_v}, \Omega_{pq})}{\partial \epsilon_{M_{ij}}} \\ s^{eq} = \frac{\partial \Psi_e(\epsilon_{M_{pq}}, \epsilon_{S_v}, \Omega_{pq})}{\partial \epsilon_{S_v}} \end{cases} \quad (13)$$

$\Psi_e$  may be expressed in a very general form:

$$\Psi_e(\epsilon_{M_{pq}}, \epsilon_{S_v}, \Omega_{pq}) = \frac{1}{2} \epsilon_{M_{ji}} D_{e_{ijkl}}(\Omega_{pq}) \epsilon_{M_{lk}} + \frac{1}{2} \epsilon_{S_v} \beta_s(\Omega_{pq}) \epsilon_{S_v} \quad (14)$$

Following the approach described in section 2, the stress fields of the real mechanical state may be described as:

$$\begin{cases} \sigma''_{ij} = \sigma^{eq}_{ij} - \tilde{\tau}_{M_{ij}} = \sigma^{eq}_{ij} - g_M \Omega_{ij} \\ s \delta_{ij} = s^{eq} \delta_{ij} - \tilde{\tau}_{S_{ij}} = s^{eq} \delta_{ij} - g_S \Omega_{ij} \end{cases} \quad (15)$$

$g_M$  and  $g_S$  are scalar material parameters, with the dimension of a rigidity. Equations 13, 14 and 15 provide the following partial derivatives of the free energy:

$$\begin{cases} \sigma''_{ij} = \frac{\partial \Psi_s(\epsilon_{M_{pq}}, \epsilon_{S_v}, \Omega_{pq})}{\partial \epsilon_{M_{ij}}} = D_{e_{ijkl}}(\Omega_{pq}) \epsilon_{M_{lk}} - g_M \Omega_{ij} \\ s = \frac{\partial \Psi_s(\epsilon_{M_{pq}}, \epsilon_{S_v}, \Omega_{pq})}{\partial \epsilon_{S_v}} = \beta_s(\Omega_{pq}) \epsilon_{S_v} - \frac{g_S}{3} \Omega_{ij} \delta_{ji} \end{cases} \quad (16)$$

The postulated expression of the free energy is defined more or less a constant, which may be set to zero. Without any additional assumption on the derivative of the free energy to damage, the following expression may thus be used:

$$\Psi_s(\epsilon_{M_{pq}}, \epsilon_{S_v}, \Omega_{pq}) = \frac{1}{2} \epsilon_{M_{ij}} D_{e_{ijkl}}(\Omega_{pq}) \epsilon_{M_{lk}} + \frac{1}{2} \epsilon_{S_v} \beta_s(\Omega_{pq}) \epsilon_{S_v} - g_M \Omega_{ij} \epsilon_{M_{ji}} - \frac{g_S}{3} \Omega_{ij} \delta_{ji} \epsilon_{S_v} \quad (17)$$

The damage-conjugated stress  $\mathbf{Y}_d$  can now be computed:

$$Y_{d_{ij}} = - \frac{\partial \Psi_s(\epsilon_{M_{pq}}, \epsilon_{S_v}, \Omega_{pq})}{\partial \Omega_{ij}} = - \frac{1}{2} \epsilon_{M_{mn}} \frac{\partial D_{e_{nmij}}(\Omega_{kl})}{\partial \Omega_{qp}} \epsilon_{M_{pq}} - \frac{1}{2} \epsilon_{S_v} \frac{\partial \beta_s(\Omega_{kl})}{\partial \Omega_{ij}} \epsilon_{S_v} + g_M \epsilon_{M_{ij}} + \frac{g_S}{3} \epsilon_{S_v} \delta_{ij} \quad (18)$$

Expression 17 is an extension of the expression used by Dragon and Halm for dry materials [8]. The proposed frame is more general. Instead of depending on two material parameters (denoted  $\alpha$  and  $\beta$  in the models of Dragon), each damaged rigidity is computed by introducing the concept of damaged stress state variable (section 4). The incremental inelastic strains are computed by deriving stress/strain relations 16 and combining them with equations 5 and 11:

$$\begin{cases} d\epsilon_{M_{ij}}^d = \left[ -\mathbf{D}_e(\Omega)^{-1} : \frac{\partial \mathbf{D}_e(\Omega)}{\partial \Omega} : \epsilon_M + g_M \mathbf{D}_e(\Omega)^{-1} \right] : d\Omega \\ d\epsilon_{S_v}^d = \left[ -\frac{\epsilon_{S_v}}{\beta_s(\Omega)} \frac{\partial \beta_s(\Omega)}{\partial \Omega} + \frac{g_S}{3\beta_s(\Omega)} \delta \right] : d\Omega \end{cases} \quad (19)$$

According to the assumption of tensile cracking (section 2), only the part of the damage-conjugated stress which is related to tensile residual strains is involved in the damage yield function [8]:

$$f_d(Y_{d_{ij}}, \Omega_{ij}) = \sqrt{\frac{1}{2} Y_{d_{ij}}^+ Y_{d_{ji}}^+} - C_0 - C_1 \delta_{ij} \Omega_{ji} \quad (20)$$

in which:

$$Y_{d_{ij}}^+ = g_M \epsilon_{M_{ij}}^+ + \frac{g_S}{3} \epsilon_{S_v}^+ \delta_{ij} \quad (21)$$

$C_0$  is the initial damage-stress rate that is necessary to trigger damage.  $C_1$  controls the damage increase rate. The damage increment is computed by an associative flow rule, which completes the expressions of the incremental inelastic strains 19.

## 4. Micro-mechanical aspects of the damage model

### 4.1. Damaged rigidities

The damaged material is assumed to have lost a part of its effective surface. Mechanically, applying the real stress fields to reduced material surfaces is thus equivalent to applying “damaged stress fields” on intact surfaces. “Damaged stress variables”  $\widehat{\sigma}_{ij}$  (named *effective stress variables* in Continuum Damage Mechanics) are defined by using a fourth-order operator depending on damage:  $\widehat{\sigma}_{ij} = M_{ijkl}(\Omega_{pq}) \sigma_{lk}$ . In the proposed model, the concept of damaged stress is extended to both independent stress state variables, by using the operator of Cordebois and Sidoroff [9]:

$$\begin{cases} \widehat{\sigma}_{ij} = (\delta - \Omega)_{ik}^{-1/2} \sigma_{kl} (\delta - \Omega)_{lj}^{-1/2} \\ \widehat{s} = \frac{s}{3} (\delta - \Omega)_{kl}^{-1} \delta_{lk} \end{cases} \quad (22)$$

The damaged rigidities  $D_{eijkl}(\Omega_{pq})$  and  $\beta_s(\Omega_{pq})$  are computed by applying the Principle of Equivalent Elastic Energy (PEEE) [10]. The PEEE postulates that the elastic deformation energy of the damaged material subjected to the real stress equals the elastic deformation energy of the corresponding fictitious intact material subjected to the damaged stress. Applying this principle to each of the two elastic potentials involved in the expression of the elastic energy  $\Psi_e$  (equation 14),  $\frac{1}{2} \epsilon_{Mij} D_{eijkl}(\Omega_{pq}) \epsilon_{Mlk}$  and  $\frac{1}{2} \epsilon_{Sv} \beta_s(\Omega_{pq}) \epsilon_{Sv}$ , leads to the following expressions for the damaged rigidities:

$$\begin{cases} D_{eijkl}(\Omega_{rs}) = M^{-1}_{ijnm}(\Omega_{rs}) D^0_{enmpq} M^T_{qpkl}(\Omega_{rs}) \\ \beta_s(\Omega_{rs}) = \frac{9\beta_s^0}{[(\delta - \Omega)^{-1}_{ij} \delta_{ji}]^2} \end{cases} \quad (23)$$

in which  $M_{ijkl}(\Omega_{rs})$  stands for the operator of Cordebois and Sidoroff used in expressions 22. As stated in [4], the use of the operator of Cordebois and Sidoroff ensures that  $\mathbf{D}_e(\boldsymbol{\Omega})$  is symmetric and positive definite, and that it is an isotropic function of the damage tensor.

### 4.2. Transfer rules

Liquid water transfer is assumed to be diffusive. The water relative velocity  $\mathbf{V}_w$  thus follows Darcy’s law, which depends on a permeability tensor  $\mathbf{K}_w$ . This conductivity tensor is split in a relative part ( $k_R$ ) related to capillary effects, and in an intrinsic part ( $\mathbf{K}_{int}$ ) related to the solid matrix [7]. Only this latter component may depend on damage:

$$\mathbf{V}_w = -\mathbf{K}_w \cdot \nabla \left( \frac{p_w}{\gamma_w} + z \right) = -k_R(S_w) \mathbf{K}_{int}(n, \Omega_{pq}) \cdot \nabla \left( \frac{p_w}{\gamma_w} + z \right) \quad (24)$$

in which  $\gamma_w$  is the volumetric weight of liquid water and  $z$  is the elevation. The intrinsic liquid water permeability is split in a reversible part and an irreversible component. The first one quantifies water flow in the reversibly damaged porous matrix, and the second one ( $\mathbf{k}_2(n^{frac}, \Omega_{rs})$ ) controls the flow in the meso-crack network:

$$K_{intij}(n, \Omega_{pq}) = k_{w0} 10^{\alpha_w e^{rev}} \delta_{ij} + k_{2ij}(n^{frac}, \Omega_{pq}) \quad (25)$$

$k_{w0}$  is the reference water permeability of the saturated isothermal porous medium (in  $m.s^{-1}$ ), and  $e^{rev}$  is the void ratio of the reversibly damaged porous material. Following the approach of Shao’s

research team [3],  $\mathbf{k}_2(n^{frac}, \Omega_{pq})$  is computed by assuming that the flow in each micro-crack is laminar. The flow is then homogenized in order to evaluate water transfers in the meso-cracks damaging the REV:

$$k_{2ij}(n^{frac}, \Omega_{pq}) = \frac{\pi^{-2/3} \gamma_w}{12 \mu_w(T_{ref})} \chi^{4/3} b^2 \sum_{k=1}^3 (d^k)^{5/3} (\delta_{ij} - n_i^k n_j^k) \quad (26)$$

$b$  plays the role of an internal length parameter, and may be determined if  $\mathbf{k}_2(n^{frac}, \Omega_{pq})$  is known for a certain damage state, which can be measured experimentally.

Air flow is also assumed to be diffusive:

$$\mathbf{V}_a = -\mathbf{K}_a \cdot \nabla \left( \frac{p_a}{\gamma_a} + z \right) \quad (27)$$

The flow is assumed to be too fast to be oriented by the fracture network. The expression of the air permeability tensor  $\mathbf{K}_a$  is thus kept unchanged from the intact configuration [7]:

$$K_{a_{ij}} = c_a \frac{\gamma_a}{\mu_a} [e(1 - S_w)]^{\alpha_a} \delta_{ij} \quad (28)$$

$\mu_a$  is the dynamic viscosity of gaseous air.  $c_a$  and  $\alpha_a$  are material parameters.  $\mathbf{K}_a$  depends on the void ratio  $e$ , and thus on total volumetric strains, which encompass an inelastic component. As a result, the effect of damage is taken into account, though considered isotropic.

## 5. Conclusion

In this paper, phenomenological and micro-mechanical concepts are used in order to develop a damage model for unsaturated porous media. The model is formulated in net stress and suction independent state variables. Correspondingly, the strain tensor is split in a mechanical part and a capillary component. The damage variable is the second-order homogenized crack density tensor. Frictional sliding on crack faces is taken into account without requiring to plasticity. The residual strains induced by crack opening are governed by a homogenized crack-related stress, which is assumed to have the same principal directions as damage. The damaged material may be represented in an equivalent mechanical stress in which it is subjected to the far field stress and to this homogenized crack-related stress. Such a representation leads to assume that Helmholtz free energy may be written as the sum of degraded elastic energies and of residual strain potentials. The incremental inelastic strains depend on the increment of damage, which is determined by an associative flow rule. The degraded rigidities are computed by applying the Principle of Equivalent Elastic Energy to each of the elastic potentials involved in the definition of the free energy. An internal length parameter is introduced in order to assess the cracking effects on liquid water permeability. Air flow is assumed to be subjected to an isotropic influence of damage, through a dependency of air permeability on volumetric strains. The representation of damage-induced anisotropy is limited to the configurations in which stress and damage have the same principal directions. The model could thus be improved in order to take into account the possible rotation of the principal crack directions. Swoboda and Yang used a conjugate-force-based damage evolution law [4]. Desmorat advocates a Kelvin decomposition of the compliance tensor [11].

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